

Reggeon-gluon vertices with Ward identities

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Abstract

Ward identities for reggeons are studied in the framework of effective action approach to the QCD in Regge kinematics. It is shown that they require introduction of new contributions not present in the reggeon diagrams initially. Application to vertices $RR \rightarrow RP$ and $RR \rightarrow RRP$ are considered and diagrams which have to be added to the QCD ones are found.

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1 Introduction

Strong interactions at high energies in the framework of the perturbative QCD in Regge kinematics can be described in terms of reggeized gluons ("reggeons"), which combine into colourless pomerons exchanged between colliding hadrons. Reggeons and their interactions were first introduced in the dispersion approach using multiple unitarity cuts [1–4]. Later, mostly to describe next order contributions, a convenient and powerful method of effective action was proposed in which the reggeons figure as independent dynamical fields interacting with the standard gluons [5, 6]. The effective action allows to present scattering amplitudes in the Regge kinematics as a sum of diagrams, similar to the Feynman ones with certain rules for propagators and interaction vertices [7]. The latter, apart from the standard QCD vertices, include the so-called induced vertices in which the reggeons interact with two or more gluons. With the growth of the number of participants the number of such diagrams grows and their form becomes more complicated. In [8] the vertex for emission of a gluon in interaction of two reggeons was calculated. It contained quite a number of terms. In the light-cone variables separate terms, as function of the longitudinal variables, were found to fall not fast enough to perform subsequent integration necessary for construction of physical amplitude, although the total vertex had a good property in this respect.

In search for a method to improve convergence in [9] Ward identities for reggeon amplitudes were proposed, which later were used to calculate the next order kernel for the interaction of three reggeons in the odderon configuration [10]. These Ward identities were based on the comparison of the diagrams including a reggeon with the ones in which this reggeon is replaced by the gluon. Obviously, the latter diagrams should not contain the ones with induced vertices for this particular reggeon. Thus the set of diagrams used for the Ward identities turned out to be narrower than the initial set of reggeon diagrams. Use of Ward identities in [10] allowed to exclude nearly all diagrams with induced vertices and achieve better convergence for the remaining ones. A brief review of the effective action and Ward identities introduced in [9] is presented in the next section.

In this note we study this problem in a more general context, having in mind colour configurations different from the simple odderon one. We show that generally not only the diagrams with the induced vertices for a particular reggeons are to be excluded from the Ward

identity for it, but also some new diagrams have to be added, which did not exist initially in the reggeon perturbation theory. It turns out that in this general case the use of Ward identities, apart from the diagrams with the QCD vertices, a whole set of special additional diagrams has to be included to correctly describe the amplitudes. In the highly symmetric odderon configuration most of these additional diagrams indeed cancel and the simple result of [10] follows in the particular gauge used there. However, in more general configurations or gauges the additional diagrams are found to be much more numerous and complicated, which puts the advantage of using Ward identities in some doubt.

We start our study in Section 3 with a simple case of the vertex $RR \rightarrow RP$ for fusion of two reggeons into one with emission of a real gluon. This vertex served as an intermediate tool for the derivation of the second order odderon interaction in [10]. So we compare this symmetric case with our general one in Section 4. In Section 5 we consider a more complicated vertex $RR \rightarrow RRP$ for emission of a gluon in interaction of two reggeons to see how our results change with the number of participating reggeons. The results of this section together with those in Section 3 clearly show the general pattern valid for arbitrary number of reggeons.

2 Effective action and Ward identities for the vertices

The effective action formalism considers interaction at a given rapidity. Apart from the normal gluon ("particle") field v an independent reggeon field A is introduced, which interacts with gluon via the so-called induced vertices and connects particles with large rapidity distance. The Lagrangian (at given rapidity) is [5, 6]:

$$\mathcal{L}_{eff} = \mathcal{L}_{QCD}(v) + 2\text{Tr} \left\{ \mathcal{V}_+(v_+) \partial_\perp^2 A_- + \mathcal{V}_-(v_-) \partial_\perp^2 A_+ \right\}. \quad (1)$$

Here $\mathcal{L}_{QCD}(v)$ is the standard QCD Lagrangian for the gluon field. In the second, "induced", term a_\pm denote \pm components of a 4-vector a in the light-cone metric: $a_\pm = (a_0 \pm a_3)/\sqrt{2}$ and

$$\begin{aligned} \mathcal{V}_\pm(v_\pm) &= -\frac{1}{g} \partial_\pm \frac{1}{D_\pm} \partial_\pm * 1 = \sum_{n=0}^{\infty} (-g)^n v_\pm (\partial_\pm^{-1} v_\pm)^n \\ &= v_\pm - g v_\pm \partial_\pm^{-1} v_\pm + g^2 v_\pm \partial_\pm^{-1} v_\pm \partial_\pm^{-1} v_\pm - \dots \end{aligned} \quad (2)$$

The reggeon fields A_+ and A_- describe incoming and outgoing reggeons respectively. In accordance with Regge kinematics they satisfy

$$\partial_- A_+ = \partial_+ A_- = 0. \quad (3)$$

In the momentum space these conditions transform into the requirement that the momentum of the incoming (outgoing) reggeon has its minus (plus) component equal to zero.

In perturbation theory the effective Lagrangian (1) generates diagrams, which apart from the standard QCD ones include those with induced vertices describing interaction of a reggeon with one or several gluons according to (2). The rules for constructing these vertices and an explicit form of some of the lowest ones was presented in [7]. The simplest of them is the vertex V_0 for transition of a reggeon into the gluon $A_\pm^a \rightarrow v_\mu^b$

$$V_0 = iQ^2 n_\mu^\mp \delta^{ab}, \quad (4)$$

where a and b are colour indices, Q is the reggeon momentum (obviously conserved in the transition) and n^\mp are pure longitudinal unit vectors: $n_+^\mp = n_-^\pm = 1$, $n_\perp^\pm = 0$. Vertices for transition into more gluons necessary for our results will be presented later.

Note that, due to the form of V_0 , transition of a reggeon into the gluon makes the diagram identical with the one in which the reggeon is replaced by the gluon. The only difference is that the reggeon polarization vector is n^- for the incoming reggeon and n^+ for the outgoing one and for the real gluon we have to use normal polarization vectors e , which also contain transverse components. This observation was the basis of the derivation of the Ward identities for reggeon vertices. In the general case, the amplitude is given by product of the vertex Γ for interaction of k gluons and l reggeons (incoming and outgoing) with their appropriate polarization vectors:

$$\mathcal{A} = e_{\mu_1}^1 \cdots e_{\mu_k}^k \cdot n_{\alpha_1}^1 \cdots n_{\alpha_l}^l \Gamma_{\alpha_1 \dots \alpha_l \mu_1 \dots \mu_k} \cdot$$

Now consider the standard Ward identity for the vertex $\bar{\Gamma}$ for interaction of $k + m$ gluons and $l - m$ reggeons, where polarization vectors of m gluons are replaced by their momenta Q_i which are supposed equal to momenta of reggeons $1 \dots m$ in Γ :

$$e_{\mu_1}^1 \cdots e_{\mu_k}^k \cdot (Q_1)_{\alpha_1} \cdots (Q_m)_{\alpha_m} \cdot n_{\alpha_{m+1}}^{m+1} \cdots n_{\alpha_l}^l \bar{\Gamma}_{\alpha_1 \dots \alpha_l \mu_1 \dots \mu_k} = 0. \quad (5)$$

In [9] it was noted that if one drops all diagrams for Γ in which reggeons $1 \dots m$ interact with two, three and more gluons and only the simple transition into a single gluon is left, then the remaining set of diagrams will be identical to the one for $\bar{\Gamma}$. So the Ward identity (5) becomes a relation between a certain subset of diagrams of the amplitude \mathcal{A} .

In [9] it was shown that application of these Ward identities allows, first, to improve convergence of the effective action diagrams at large longitudinal momenta and, second, may hopefully eliminate at least some of the diagrams with induced vertices altogether. This hope was later confirmed in the calculation of the next order corrections to the odderon kernel in [10], where all but one induced diagrams for the vertex $RR \rightarrow RP$ were eliminated after multiple application of the Ward identities. In the next section we shall demonstrate that this result in fact refers to the particular configuration and chosen gauge in the study of [10]. In a more general configuration and gauge the reduction in the number of diagrams with induced vertices is more modest.

3 Vertex $RR \rightarrow RP$

3.1 Notations and diagrams

We study the amplitude for gluon production \mathcal{A} in the fusion of two reggeons into one

$$R_{a_2}(Q_2) + R_{a_1}(Q_1) \rightarrow R_b(R) + g_c(p), \quad (6)$$

where a_2, a_1, b and c are colour indices. This amplitude is a product of vertex Γ for the transition $RR \rightarrow RP$ with the gluon polarization vector e

$$\mathcal{A} = (e\Gamma) = e_\mu \Gamma_\mu. \quad (7)$$

In the following multiplication by e will in many cases not be written explicitly. We use the standard light-cone metric and only covariant components. Summation over repeated 4-vector indices is assumed in the Lorentz light-cone metric. The amplitude splits into two independent parts with colour indices $C_1 = f^{a_2 cd} f^{ba_1 d}$ and $C_2 = f^{a_1 cd} f^{ba_2 d}$. We study the part with the colour vertex C_1 here. The other part can be studied in quite the similar manner.

As explained in the previous section, we introduce polarization vectors for reggeons: n^- for the incoming reggeons with momenta Q_1 and Q_2 and n^+ for the outgoing reggeon with

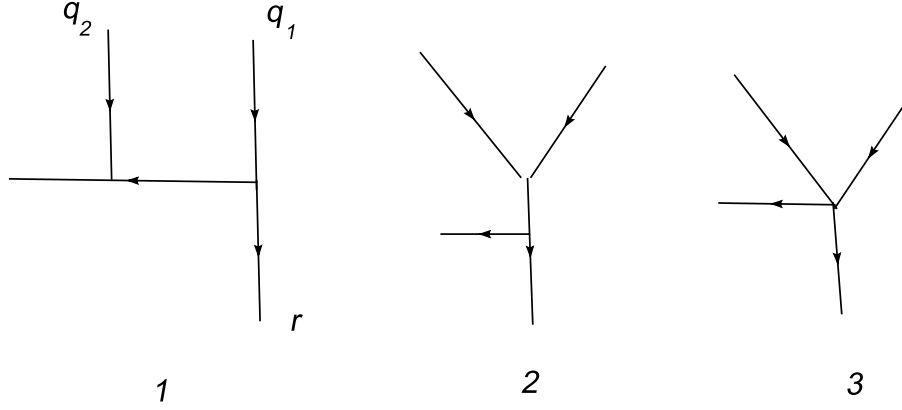


Figure 1: Diagrams for the vertex $RR \rightarrow RP$ with QCD vertices. Simple external lines directed upwards and downwards actually refer to reggeons.

momentum R . To simplify notation in the following we denote the outgoing reggeon by index 3, so that $Q_3 \equiv R$. Then vertex $RR \rightarrow RP$ will be given by the convolution

$$\Gamma_\mu = (n_1 n_2 n_3 D)_\mu \equiv n_{\alpha_1}^- n_{\alpha_2}^- n_\beta^+ D_{\alpha_1 \alpha_2 \beta \mu}, \quad (8)$$

where D is the sum of 9 diagrams shown in Figs. 1 and 2. In these diagrams it is assumed that simple external lines directed upwards and downwards actually refer to reggeons, which go into gluons via vertex V_0 (Eq. (4)). They include purely QCD diagrams D_0 (Fig. 1) and part of the diagrams with induced vertices for reggeons 1,2 and 3 (Fig. 2)

$$D = D_0 + I_{1,1} + I_{2,1} + I_3 + I_{23,1} \quad (9)$$

(9 diagrams in all). Here I_i is the sum of diagrams with only a single induced vertex for reggeon i , $I_{i,k}$ is the k th diagram for I_i . Likewise I_{ik} is the sum of diagrams with two induced vertices for reggeons i and k and $I_{ik,l}$ is the l th diagram for this sum in Fig. 2.

Since the induced vertex for transition of the reggeon into gluon is trivial, a simple external line in Figs. 1 and 2 is just unity and vertices connecting simple lines are the standard QCD vertices. As to the vertices for transition of the reggeon into 2 or 3 gluons, they are shown in Fig. 3 and given by (see [7])

$$\begin{aligned} V_1 &= gq^2 f^{acb} \frac{1}{l_-} n_\lambda^- n_\beta^- n_\alpha^+, \\ V_2 &= -gq^2 f^{acb} \frac{1}{l_+} n_\lambda^+ n_\beta^+ n_\alpha^-, \\ V_3 &= ig^2 q^2 n_\lambda^- n_\mu^- n_\beta^- n_\alpha^+ \left(\frac{f^{bce} f^{dea}}{m_- l_-} + \frac{f^{dce} f^{bea}}{k_- l_-} \right), \\ V_4 &= ig^2 q^2 n_\lambda^+ n_\mu^+ n_\beta^+ n_\alpha^- \left(\frac{f^{bce} f^{dea}}{m_+ l_+} + \frac{f^{dce} f^{bea}}{k_+ l_+} \right). \end{aligned} \quad (10)$$

The meaning of indices s , (0) , $(+)$ and $(-)$ will be explained later.

We are going to use the Ward identities introduced in [9] for the calculation of the amplitude \mathcal{A} . As explained in the previous section, they are based on the comparison of the reggeon diagrams with the ones where a particular reggeon is replaced by the gluon. In such diagrams this particular reggeon obviously cannot be coupled with induced vertices with more than one gluon. So these Ward identities actually should be applied only to a

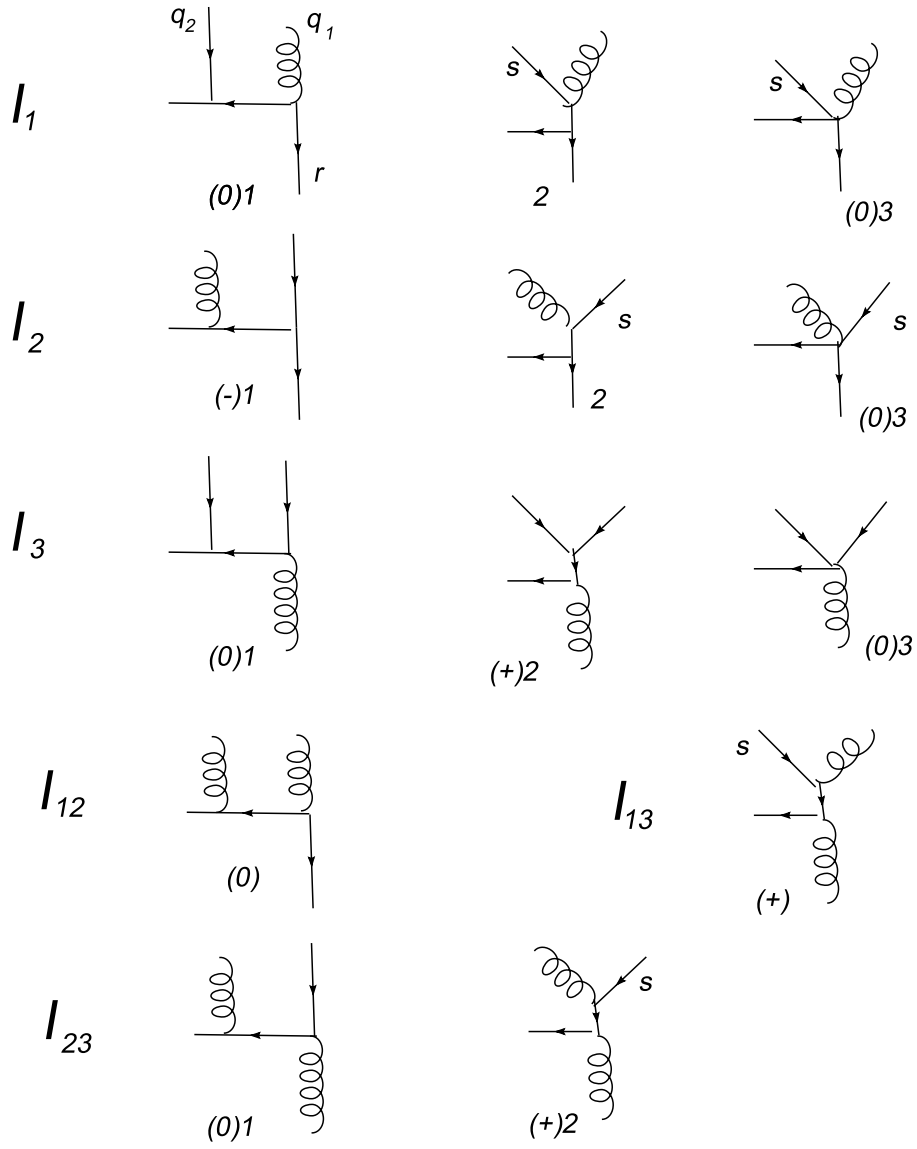


Figure 2: Diagrams for the vertex $RR \rightarrow RP$ and Ward identities with induced vertices. Simple external lines directed upwards and downwards actually refer to reggeons.

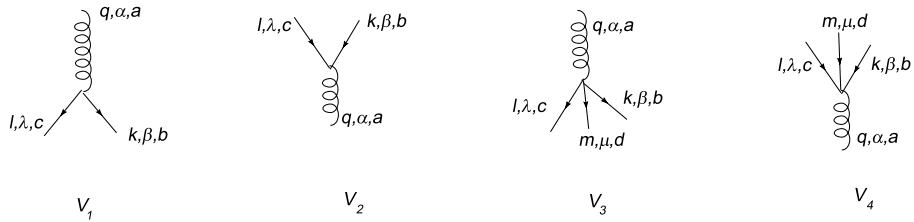


Figure 3: Induced vertices for transition of the reggeon into gluons

part of all contributions which does not contain higher induced vertices for the considered reggeon. However, this is not the end of the story. In fact, as we shall see, the amplitude with the reggeon replaced by the gluon may contain additional diagrams, which are absent for the reggeon. Such diagrams do not give contribution for the amplitude \mathcal{A} but do give contributions to the Ward identities.

In our case these additional diagrams are diagrams in Fig. 2 with a single induced vertex $I_{i,2}$ and $I_{i,3}$ where $i = 1, 2$ and with two induced vertices I_{13} and $I_{23,2}$.

One observes that some of the diagrams with induced vertices may in fact give zero for the vertex Γ after multiplication by the product $n^- n^- n^+$ in (8). In our case among the initial reggeon diagrams $I_{3,2}$ vanishes in (8) due to the property of the three-gluon vertex in the Regge kinematics.

Also the diagrams in Fig. 2 added to the initial ones to cover application of Ward identities vanish in (8) for the same reason. However, one should take certain care with these diagrams.

It is instructive to study their explicit form. As an example, take $I_{1,2}$ with a single induced vertex for reggeon 1. It is given by

$$I_{1,2} = -n_{\alpha_1}^+ n_{\alpha_2}^- n_{\nu}^- \frac{Q_1^2}{Q_{2-}(Q_1 + Q_2)^2} \gamma_{\nu\beta\mu}(Q_1 + Q_2, -R), \quad (11)$$

where $\gamma_{\mu\nu\sigma}(Q, R)$ is the three-gluon vertex for incoming momenta Q_μ, R_ν and $-(Q + R)_\sigma$. Remarkably $I_{1,2}$ contains Q_{2-} in the denominator and has no sense when gluon 2 becomes the reggeon with $Q_{2-} = 0$. So in fact multiplied by the reggeon 2 polarization vector $n_{\alpha_2}^-$, it contains an indefinite factor $(n^-)^2/Q_{2-}$. To be able to include such a diagram into the general form (8) and (9) we have to impose the condition

$$n_{\alpha_2}^- [I_{1,2}]_{\alpha_1\alpha_2\beta\mu} = 0$$

equivalent to the interpretation

$$\left. \frac{n^-^2}{Q_{2-}} \right|_{Q_{2-}=0} = 0. \quad (12)$$

Then this diagram will not appear in the expression for (8) for Γ and we can include it as a singular contribution into I_1 . However, it gives a non-zero contribution to the Ward identity for reggeon 2. Indeed, multiplication by scaled Q_2/Q_{2+} replaces Q_{2-} by Q_{2+} in the denominator and makes it perfectly valid for the Regge kinematics with $Q_{2-} = 0$. The same is true for all the rest of similar diagrams in Fig. 2. To indicate appearance of such singular denominators we mark the corresponding external legs by letter s in Fig. 2.

So we conclude that to apply Ward identities it is not sufficient to drop the diagrams with transition of a chosen reggeon into two or more gluons. One has also to add some new diagrams, which are absent for the diagram with the chosen reggeon but are present when it is replaced by the gluon.

3.2 Ward identities

In our derivation to simplify notations we normalize the incoming reggeon momenta putting

$$\frac{Q_i}{Q_{i+}} = q_i, \quad i = 1, 2$$

and for the outgoing reggeon

$$\frac{R}{R_-} \rightarrow q_3.$$

So we denote momenta of all reggeons, incoming and outgoing, by just q_1, q_2, q_3 and their polarization vectors as n_1, n_2, n_3 having in mind that for the incoming reggeons we have to take n^- and for the outgoing reggeons n^+ . We denote the transverse momenta of all reggeons, duly normalized, by t_1, t_2, t_3 , so that in these notations

$$n = q - t. \quad (13)$$

Using (13) we rewrite (8) as

$$\Gamma = (q_1 - t_1)(q_2 - t_2)(q_3 - t_3)D. \quad (14)$$

We find 8 terms. Taking into account that induced vertices are longitudinal we find these terms as

$$\begin{aligned} a_1 &= q_1 q_2 q_3 D, \\ a_2 &= -q_1 q_2 t_3 (D_0 + I_1 + I_2 + I_{12}), \\ a_3 &= -q_1 t_2 q_3 (D_0 + I_1 + I_3 + I_{13}), \\ a_4 &= -t_1 q_2 q_3 (D_0 + I_2 + I_3 + I_{23}), \\ a_5 &= q_1 t_2 t_3 (D_0 + I_1), \\ a_6 &= t_1 t_2 q_3 (D_0 + I_3), \\ a_7 &= t_1 q_2 t_3 (D_0 + I_2), \\ a_8 &= -t_1 t_2 t_3 D_0. \end{aligned}$$

We want to calculate these term using Ward identities (WI)

$$q_1 n_2 n_3 (D_0 + I_2 + I_3 + I_{23}) = 0, \quad (15)$$

$$q_1 q_2 n_3 (D_0 + I_3) = 0, \quad (16)$$

4 similar ones obtained after permutations (123)→(231) and (123)→(312) and

$$q_1 q_2 q_3 D_0 = 0. \quad (17)$$

Our strategy will be to express from WI the result of contraction of momenta q_i with D_0 . Take (15). It reads

$$q_1 (q_2 - t_2)(q_3 - t_3)(D_0 + I_2 + I_3 + I_{23}) = 0 \quad (18)$$

or

$$q_1 t_2 t_3 D_0 = q_1 t_2 q_3 (D_0 + I_3) + q_1 q_2 t_3 (D_0 + I_2) - q_1 q_2 q_3 (I_2 + I_3 + I_{23}), \quad (19)$$

where we used (17). We put it into a_5

$$a_5 = q_1 t_2 t_3 I_1 + q_1 t_2 q_3 I_3 + q_1 q_2 t_3 I_2 - q_1 q_2 q_3 (I_2 + I_3 + I_{23}) + (q_1 t_2 q_3 + q_1 q_2 t_3) D_0. \quad (20)$$

Now we use (16)

$$q_1 q_2 (q_3 - t_3)(D_0 + I_3) = 0,$$

which with (17) taken into account gives

$$q_1 q_2 t_3 D_0 = q_1 q_2 q_3 I_3. \quad (21)$$

Smilarly,

$$q_1 t_2 q_3 D_0 = q_1 q_2 q_3 I_2. \quad (22)$$

Put into (20) it changes the last term into $q_1 q_2 q_3 (I_2 + I_3)$ and cancels two first term in the preceding term. We get

$$a_5 = q_1 t_2 t_3 I_1 + q_1 t_2 q_3 I_3 + q_1 q_2 t_3 I_2 - q_1 q_2 q_3 I_{23} . \quad (23)$$

Similarly,

$$a_6 = q_3 t_2 t_1 I_3 + q_3 t_2 q_1 I_1 + q_3 q_2 t_1 I_2 - q_1 q_2 q_3 I_{12} , \quad (24)$$

$$a_7 = q_2 t_1 t_3 I_2 + q_2 t_1 q_3 I_3 + q_2 q_1 t_3 I_1 - q_1 q_2 q_3 I_{13} . \quad (25)$$

Now take a_2 and use (21)

$$a_2 = -q_1 q_2 t_3 (I_1 + I_2 + I_{12}) - q_1 q_2 q_3 I_3 .$$

Similarly,

$$a_3 = -q_1 t_2 q_3 (I_1 + I_3 + I_{13}) - q_1 q_2 q_3 I_2 ,$$

$$a_4 = -t_1 q_2 q_3 (I_2 + I_3 + I_{23}) - q_1 q_2 q_3 I_1 .$$

In the sum

$$\begin{aligned} \sum_{i=2}^7 a_i &= q_1 t_2 t_3 I_1 + q_3 t_2 t_1 I_3 + q_2 t_1 t_3 I_2 - q_1 q_2 t_3 I_{12} - q_1 t_2 q_3 I_{13} - t_1 q_2 q_3 I_{23} \\ &\quad - q_1 q_2 q_3 (I_1 + I_2 + I_3 + I_{12} + I_{13} + I_{23}) . \end{aligned}$$

The last term cancels a_1 and we finally get

$$\Gamma_{RR \rightarrow RP} = -t_1 t_2 t_3 D_0 + q_1 t_2 t_3 I_1 + t_1 q_2 t_3 I_2 + t_1 t_2 q_3 I_3 - q_1 q_2 t_3 I_{12} - q_1 t_2 q_3 I_{13} - t_1 q_2 q_3 I_{23} . \quad (26)$$

Here factors q_i applied to reggeons from induced vertices can be replaced by n_i , since reggeon legs are longitudinal.

At this moment it is necessary to take into account condition (12) for the singular parts of contributions from Fig. 2 with index s at some external legs. Condition (12) allows to substitute in such diagrams $t \rightarrow q$ for this leg, which cancels the singularity due to the denominator.

4 Additional terms in \mathcal{A}

The attractive feature of application of Ward identities to the calculation of high energy amplitudes represented by reggeon diagrams is that in the framework of effective action it allows, at least partially, to exclude contributions from induced vertices leaving only QCD diagrams convoluted with transverse momenta. In our case such QCD contribution is given by term $-t_1 t_2 t_3 D_0$. However, Eq. (26) shows that in fact to the QCD contribution certain terms have to be added, which include diagrams with induced vertices and moreover some diagrams absent at all in the reggeon diagram technique. In this section we study this additional contributions in Eq. (26). As we shall find many of the additional terms are in fact zero. However not all, so that the QCD term alone cannot give the correct value for the amplitude (on mass shell, multiplied by the polarization vector e).

To locate diagrams which actually vanish we note that all external lines which eventually transform into reggeons and do not bear index s are to be multiplied by the appropriate transverse vector t . They can be attached either to QCD vertices or to induced ones. In the latter case multiplication by the transverse vector t will give zero, as is evident from rules (10). This removes the contribution from all diagrams in Fig. 2 marked by index (0). Then we are left with only 6 diagrams $I_{1,2}$, $I_{2,1}$, $I_{2,2}$, $I_{3,2}$, I_{13} and $I_{23,2}$.

Some further reduction of the number of contributing diagrams can be achieved choosing special gauges with either $e_+ = 0$ or $e_- = 0$. In fact the line corresponding to the emitted gluon can also be attached either to the QCD vertex or to the induced one. In the latter case it bears factor n^\pm depending on which induced vertex it is attached to. In Fig. 2 such diagram is marked by index (\pm) . It is clear that all diagrams marked by $(+)$ will vanish in the gauge with $e_+ = 0$ and all diagrams marked by $(-)$ will vanish in the gauge $e_- = 0$. In particular, in the commonly assumed gauge $e_+ = 0$ the only remaining additional diagrams are $I_{1,2}$, $I_{2,1}$ and $I_{2,2}$.

In the following for illustration we calculate all additional contributions which are non-zero in the general gauge.

Term $t_1 n_2 t_3 I_{2,1}$

The diagram is given by

$$I_{2,1} = -n_{\alpha_2}^+ n_\nu^- n_\mu^- \frac{Q_2^2}{T^2 R_-} \gamma_{\alpha_1 \beta \nu}(Q_1, -R) \quad (27)$$

with $T = Q_1 - R$. We find

$$t_1 n_2 t_3 I_2 = -n_\mu^- t_{1\alpha_1} t_{3\beta} n_\nu^- \gamma_{\alpha_1 \beta \nu}(Q_1, -R) \frac{Q_2^2}{T^2 R_-} \quad (28)$$

and the contribution to the amplitude \mathcal{A} will be

$$\Delta_{2,1} = -e_- t_{1\alpha_1} t_{3\beta} n_\nu^- \gamma_{\alpha_1 \beta \nu}(Q_1, -R) \frac{Q_2^2}{T^2 R_-}. \quad (29)$$

So this term is given by the product of the QCD vertex on the right convoluted with transverse momenta of reggeons 1 and 3 and multiplied by the induced vertex for reggeon 2 on the left. It is the term which was explicitly introduced in the calculations of the transition $RRR \rightarrow RRR$ in the odderon case in [10].

Term $t_1 t_2 q_3 I_{3,2}$

We have

$$I_{3,2} = -n_\nu^+ n_\beta^- n_\mu^+ \frac{R^2}{p_+ \tilde{T}^2} \gamma_{\alpha_1 \nu \alpha_2}(Q_1, -\tilde{T})$$

with $\tilde{T} = Q_1 + Q_2$. So

$$t_1 t_2 n_3 I_{3,2} = -n_\mu^+ \frac{R^2}{p_+ \tilde{T}^2} n_\nu^+ t_{1\alpha_1} t_{\alpha_2} \gamma_{\alpha_1 \nu \alpha_2}(Q_1, -\tilde{T}).$$

One has

$$n_\nu^+ \gamma_{\alpha_1 \nu \alpha_2}(Q_1, -\tilde{T}) = (Q_2 - Q_1)_+ g_{\alpha_2 \alpha_1} + (2Q_1 + Q_2)_{\alpha_2} n_{\alpha_1}^+ - (2Q_2 + Q_1)_{\alpha_1} n_{\alpha_2}^+.$$

As a consequence

$$t_1 t_2 n_3 I_{3,2} = -n_\mu^+ (Q_2 - Q_1)_+ \frac{R^2(t_1 t_2)}{p_+ \tilde{T}^2}. \quad (30)$$

This leads to the additional contribution to the amplitude

$$\Delta_{3,2} = -e_+ (Q_2 - Q_1)_+ \frac{R^2(t_1 t_2)}{p_+ \tilde{T}^2}. \quad (31)$$

So we obtain a non-zero additional contribution. Like $\Delta_{2,1}$ it is given by the product of the QCD vertex on the top convoluted with transverse momenta of reggeons 1 and 2 and

multiplied by the induced vertex for reggeon 3 on the bottom. However, it vanishes in the gauge with $e_+ = 0$. It also vanishes for the sum of contributions with the order of Q_1 and Q_2 reversed ("after symmetrization"). This is why it did not appear in [10] where such symmetrization occurred due to the odderon colour structure.

Term $n_1 t_2 t_3 I_{1,2}$

The explicit expression for $I_{1,2}$ is

$$I_{1,2} = -n_{\alpha_1}^+ n_{\nu}^- n_{\alpha_2}^- \frac{Q_1^2}{Q_{2-} \tilde{T}^2} \gamma_{\nu\beta\mu}(\tilde{T}, -R).$$

As explained one should substitute factor q_2 for t_2 which cancels the denominator Q_{2-} . Then

$$q_2 I_{1,2} = -n_{\alpha_1}^+ n_{\nu}^- \frac{Q_1^2}{Q_{2+} \tilde{T}^2} \gamma_{\nu\beta\mu}(\tilde{T}, -R),$$

so that

$$n_1 q_2 t_3 I_{1,2} = -t_{3\beta} n_{\nu}^- \frac{Q_1^2}{Q_{2+} \tilde{T}^2} \gamma_{\nu\beta\mu}(\tilde{T}, -R).$$

We have

$$\gamma_{\nu\beta\mu}(\tilde{T}, -R) = 2R_{\mu} g_{\beta\nu} + (p - R)_{\nu} g_{\beta\mu} - (p + \tilde{T})_{\beta} g_{\mu\nu} ,$$

so that

$$n_1 q_2 t_3 I_{1,2} = \left(2R_{-} t_{3\mu} + (t_3, p + \tilde{T}) n_{\mu}^- \right) \frac{Q_1^2}{Q_{2+} \tilde{T}^2} \quad (32)$$

and the additional term in the amplitude \mathcal{A} will be

$$\Delta_{1,2} = \left(2(eR)_{\perp} + e_{-} \frac{(R, p + \tilde{T})_{\perp}}{R_{-}} \right) \frac{Q_1^2}{Q_{2+} \tilde{T}^2}. \quad (33)$$

Term $t_1 n_2 t_3 I_{2,2}$

The explicit expression for $I_{2,2}$ is

$$I_{2,2} = n_{\alpha_2}^+ n_{\nu}^- n_{\alpha_1}^- \frac{Q_2^2}{Q_{1-} \tilde{T}^2} \gamma_{\nu\beta\mu}(\tilde{T}, -R).$$

We change $t_1 \rightarrow q_1$, which gives

$$q_1 I_{2,2} = n_{\alpha_2}^+ n_{\nu}^- \frac{Q_2^2}{Q_{1+} \tilde{T}^2} \gamma_{\nu\beta\mu}(\tilde{T}, -R),$$

so that

$$q_1 n_2 t_3 I_{2,2} = \left(-2R_{-} t_{3\mu} - (t_3, p + \tilde{T}) n_{\mu}^- \right) \frac{Q_2^2}{Q_{1+} \tilde{T}^2} \quad (34)$$

and the additional term in the amplitude \mathcal{A} will be

$$\Delta_{2,2} = - \left(2(eR)_{\perp} + e_{-} \frac{(R, p + \tilde{T})_{\perp}}{R_{-}} \right) \frac{Q_2^2}{Q_{1+} \tilde{T}^2}. \quad (35)$$

Term $n_1 t_2 n_3 I_{13}$

The diagram is given by

$$I_{13} = -n_{\beta}^- n_{\mu}^+ n_{\alpha_1}^+ n_{\alpha_2}^- \frac{Q_1^2 R^2}{p_{+} Q_{2-} \tilde{T}^2}.$$

Correspondingly, changing $t_2 \rightarrow q_2$,

$$q_2 I_{13} = -n_\beta^- n_\mu^+ n_{\alpha_1}^+ \frac{Q_1^2 R^2}{p_+ Q_{2+} \tilde{T}^2}$$

and

$$n_1 q_2 n_3 I_{13} = -n_\mu^+ \frac{Q_1^2 R^2}{p_+ Q_{2+} \tilde{T}^2}.$$

The additional term in the amplitude is

$$\Delta_{13} = e_+ \frac{Q_1^2 R^2}{p_+ Q_{2+} \tilde{T}^2}. \quad (36)$$

Term $t_1 n_2 n_3 I_{23,2}$

The diagram is given by

$$I_{23,2} = n_\beta^- n_\mu^+ n_{\alpha_1}^- n_{\alpha_2}^+ \frac{Q_2^2 R^2}{p_+ Q_{1-} \tilde{T}^2}.$$

Correspondingly, with the change $t_1 \rightarrow q_1$

$$q_1 I_{23,2} = n_\beta^- n_\mu^+ n_{\alpha_2}^+ \frac{Q_2^2 R^2}{p_+ Q_{1+} \tilde{T}^2}$$

and

$$q_1 n_2 n_3 I_{23,2} = n_\mu^+ \frac{Q_2^2 R^2}{p_+ Q_{1+} \tilde{T}^2}.$$

The additional term in the amplitude is

$$\Delta_{23,2} = -e_+ \frac{Q_2^2 R^2}{p_+ Q_{1+} \tilde{T}^2}. \quad (37)$$

Total additional terms

They are six

$$\Delta_{tot} = \Delta_{2,1} + \Delta_{3,2} + \Delta_{1,2} + \Delta_{2,2} + \Delta_{13} + \Delta_{23,2}. \quad (38)$$

Of these terms $\Delta_{3,2}$, Δ_{13} and $\Delta_{23,2}$ are proportional to e_+ and vanish in the gauge $e_+ = 0$. Also all contributions except $\Delta_{2,1}$ are antisymmetric in Q_1 and Q_2 . Because of this they do not appear in the odderon case.

5 Vertex $RR \rightarrow RRP$

5.1 Vertex with Ward identities

Previous results have a structure, which can naturally be generalized for larger number of reggeons. For certainty we here study the case of the vertex $RR \rightarrow RRP$.

In our notations with 4 reggeons the vertex as a 4-vector is given by

$$\left(\Gamma_{RR \rightarrow RRP} \right)_\mu = (n_1)_{\alpha_1} (n_2)_{\alpha_2} (n_3)_{\alpha_3} (n_4)_{\alpha_4} D_{\alpha_1 \alpha_2 \alpha_3 \alpha_4 \mu}, \quad (39)$$

where summation over repeated indices is understood in the Lorentz metric. For brevity, we rewrite it as

$$\Gamma_{RR \rightarrow RRP} = n_1 n_2 n_3 n_4 D. \quad (40)$$

As before, D is the sum of purely QCD diagrams D_0 and diagrams with induced vertices for reggeons 1,2,3 and 4

$$D = D_0 + \sum_{i=1}^4 I_i + \sum_{i < k=2}^4 I_{ik} + \sum_{i < k < l=3}^4 I_{ikl} , \quad (41)$$

where I_i , I_{ik} and I_{ikl} are sum of diagrams with induced vertices. Using (13) we have

$$\Gamma_{RR \rightarrow RRP} = \prod_{i=1}^4 (q_i - t_i) D. \quad (42)$$

We find 16 terms. Taking into account that induced vertices are longitudinal we find these terms as

$$\begin{aligned} a_1 &= q_1 q_2 q_3 q_4 D, \\ a_2 &= -q_1 q_2 q_3 t_4 (D_0 + I_1 + I_2 + I_3 + I_{12} + I_{13} + I_{23} + I_{123}), \\ a_3 &= -q_1 q_2 t_3 q_4 (D_0 + I_1 + I_2 + I_4 + I_{12} + I_{14} + I_{24} + I_{124}), \\ a_4 &= -q_1 t_2 q_3 q_4 (D_0 + I_1 + I_3 + I_4 + I_{13} + I_{14} + I_{34} + I_{134}), \\ a_5 &= -t_1 q_2 q_3 q_4 (D_0 + I_2 + I_3 + I_4 + I_{23} + I_{24} + I_{34} + I_{234}), \\ a_6 &= q_1 q_2 t_3 t_4 (D_0 + I_1 + I_2 + I_{12}), \\ a_7 &= q_1 t_2 q_3 t_4 (D_0 + I_1 + I_3 + I_{13}), \\ a_8 &= q_1 t_2 t_3 q_4 (D_0 + I_1 + I_4 + I_{14}), \\ a_9 &= t_1 q_2 q_3 t_4 (D_0 + I_2 + I_3 + I_{23}), \\ a_{10} &= t_1 q_2 t_3 q_4 (D_0 + I_2 + I_4 + I_{24}), \\ a_{11} &= t_1 t_2 q_3 q_4 (D_0 + I_3 + I_4 + I_{34}), \\ a_{12} &= -q_1 t_2 t_3 t_4 (D_0 + I_1), \\ a_{13} &= -t_1 q_2 t_3 t_4 (D_0 + I_2), \\ a_{14} &= -t_1 t_2 q_3 t_4 (D_0 + I_3), \\ a_{15} &= -t_1 t_2 t_3 q_4 (D_0 + I_4), \\ a_{16} &= t_1 t_2 t_3 t_4 D_0 . \end{aligned}$$

To calculate these terms we use the WI

$$q_1 n_2 n_3 n_4 (D_0 + I_2 + I_3 + I_4 + I_{23} + I_{24} + I_{34} + I_{234}) = 0 \quad (43)$$

and 3 similar ones obtained by $q_1 \rightarrow q_2, q_3, q_4$,

$$q_1 q_2 n_3 n_4 (D_0 + I_3 + I_4 + I_{34}) = 0 \quad (44)$$

and 5 similar ones obtained by $q_1 q_2 \rightarrow q_1 q_3, q_1 q_4, q_2 q_3, q_2 q_4, q_3 q_4$,

$$q_1 q_2 q_3 n_4 (D_0 + I_4) = 0 \quad (45)$$

and 3 similar ones obtained by $n_4 \rightarrow n_1, n_2, n_3$ and finally

$$q_1 q_2 q_3 q_4 D_0 = 0. \quad (46)$$

Our strategy remains the same: using WI we express contractions of momenta q_i with D_0 . We rewrite (43) as

$$q_1(q_2 - t_2)(q_3 - t_3)(q_4 - t_4)(D_0 + I_2 + I_3 + I_4 + I_{23} + I_{24} + I_{34} + I_{234}) = 0$$

and find (using (46))

$$\begin{aligned} q_1 t_2 t_3 t_4 D_0 &= q_1 q_2 q_3 q_4 (I_2 + I_3 + I_4 + I_{23} + I_{24} + I_{34} + I_{234}) \\ &- q_1 q_2 q_3 t_4 (D_0 + I_2 + I_3 + I_{23}) - q_1 q_2 t_3 q_4 (D_0 + I_2 + I_4 + I_{24}) - q_1 t_2 q_3 q_4 (D_0 + I_3 + I_4 + I_{34}) \\ &+ q_1 q_2 t_3 t_4 (D_0 + I_2) + q_1 t_2 q_3 t_4 (D_0 + I_3) + q_1 t_2 t_3 q_4 (D_0 + I_4). \end{aligned} \quad (47)$$

We put it into a_{12}

$$\begin{aligned} a_{12} &= -q_1 t_2 t_3 t_4 I_1 - q_1 q_2 q_3 q_4 (I_2 + I_3 + I_4 + I_{23} + I_{24} + I_{34} + I_{234}) \\ &+ q_1 q_2 q_3 t_4 (I_2 + I_3 + I_{23}) + q_1 q_2 t_3 q_4 (I_2 + I_4 + I_{24}) + q_1 t_2 q_3 q_4 (I_3 + I_4 + I_{34}) \\ &- q_1 q_2 t_3 t_4 I_2 - q_1 t_2 q_3 t_4 I_3 - q_1 t_2 t_3 q_4 I_4 + (q_1 q_2 q_3 t_4 + q_1 q_2 t_3 q_4 + q_1 t_2 q_3 q_4 \\ &- q_1 q_2 t_3 t_4 - q_1 t_2 q_3 t_4 - q_1 t_2 t_3 q_4) D_0. \end{aligned} \quad (48)$$

We have also similar expression for a_{13} , a_{14} and a_{15} .

Now we rewrite (44) as

$$q_1 q_2 (q_3 - t_3)(q_4 - t_4)(D_0 + I_3 + I_4 + I_{34}) = 0,$$

wherefrom we find

$$q_1 q_2 t_3 t_4 D_0 = -q_1 q_2 q_3 q_4 (I_3 + I_4 + I_{34}) + q_1 q_2 t_3 q_4 (D_0 + I_4) + q_1 q_2 q_3 t_4 (D_0 + I_3). \quad (49)$$

We rewrite (45) as

$$q_1 q_2 q_3 (q_4 - t_4)(D_0 + I_4) = 0,$$

wherefrom

$$q_1 q_2 q_3 t_4 D_0 = q_1 q_2 q_3 q_4 I_4.$$

Similarly,

$$q_1 q_2 t_3 q_4 D_0 = q_1 q_2 q_3 q_4 I_3,$$

$$q_1 t_2 q_3 q_4 D_0 = q_1 q_2 q_3 q_4 I_2,$$

$$t_1 q_2 q_3 q_4 D_0 = q_1 q_2 q_3 q_4 I_1.$$

Using this we find

$$q_1 q_2 t_3 t_4 D_0 = -q_1 q_2 q_3 q_4 I_{34} + q_1 q_2 t_3 q_4 I_4 + q_1 q_2 q_3 t_4 I_3.$$

We put this into a_6

$$a_6 = q_1 q_2 t_3 t_4 (I_1 + I_2 + I_{12}) + q_1 q_2 t_3 q_4 I_4 + q_1 q_2 q_3 t_4 I_3 - q_1 q_2 q_3 q_4 I_{34}. \quad (50)$$

Other similar formulas are

$$q_1 t_2 q_3 t_4 D_0 = -q_1 q_2 q_3 q_4 I_{24} + q_1 q_2 q_3 t_4 I_2 + q_1 t_2 q_3 q_4 I_4,$$

$$q_1 t_2 t_3 q_4 D_0 = -q_1 q_2 q_3 q_4 I_{23} + q_1 q_2 t_3 q_4 I_2 + q_1 t_2 q_3 q_4 I_3,$$

$$t_1 q_2 q_3 t_4 D_0 = -q_1 q_2 q_3 q_4 I_{14} + q_1 q_2 q_3 t_4 I_1 + t_1 q_2 q_3 q_4 I_4,$$

$$\begin{aligned}
t_1 q_2 t_3 q_4 D_0 &= -q_1 q_2 q_3 q_4 I_{13} + q_1 q_2 t_3 q_4 I_1 + t_1 q_2 q_3 q_4 I_3, \\
t_1 t_2 q_3 q_4 D_0 &= -q_1 q_2 q_3 q_4 I_{12} + q_1 t_2 q_3 q_4 I_1 + t_1 q_2 q_3 q_4 I_2.
\end{aligned}$$

Using these formulas we find

$$\begin{aligned}
q_1 t_2 t_3 t_4 D_0 &= -q_1 q_2 q_3 t_4 I_{23} - q_1 q_2 t_3 q_4 I_{24} - q_1 t_2 q_3 q_4 I_{34} \\
&\quad + q_1 q_2 t_3 t_4 I_2 + q_1 t_2 q_3 t_4 I_3 + q_1 t_2 t_3 q_4 I_4 + q_1 q_2 q_3 q_4 I_{234}, \\
t_1 q_2 t_3 t_4 D_0 &= -q_1 q_2 q_3 t_4 I_{13} - q_1 q_2 t_3 q_4 I_{14} - t_1 q_2 q_3 q_4 I_{34} \\
&\quad + q_1 q_2 t_3 t_4 I_1 + t_1 q_2 q_3 t_4 I_3 + t_1 q_2 t_3 q_4 I_4 + q_1 q_2 q_3 q_4 I_{134}, \\
t_1 t_2 q_3 t_4 D_0 &= -q_1 q_2 q_3 t_4 I_{12} - q_1 t_2 q_3 q_4 I_{14} - t_1 q_2 q_3 q_4 I_{24} \\
&\quad + q_1 t_2 q_3 t_4 I_1 + t_1 q_2 q_3 t_4 I_2 + t_1 t_2 q_3 q_4 I_4 + q_1 q_2 q_3 q_4 I_{124}, \\
t_1 t_2 t_3 q_4 D_0 &= -q_1 q_2 t_3 q_4 I_{12} - q_1 t_2 q_3 q_4 I_{13} - t_1 q_2 q_3 q_4 I_{23} \\
&\quad + q_1 t_2 t_3 q_4 I_1 + t_1 q_2 t_3 q_4 I_2 + t_1 t_2 q_3 q_4 I_3 + q_1 q_2 q_3 q_4 I_{123}.
\end{aligned}$$

This allows to exclude all terms with D_0 multiplied by q_i . Call $-X_3$ the sum of such terms in $a_{12} + a_{13} + a_{14} + a_{15}$. We find

$$\begin{aligned}
X_3 &= q_1 q_2 q_3 t_4 (I_{23} + I_{13} + I_{12}) + q_1 q_2 t_3 q_4 (I_{24} + I_{14} + I_{12}) \\
&\quad + q_1 t_2 q_3 q_4 (I_{34} + I_{14} + I_{13}) + t_1 q_2 q_3 q_4 (I_{34} + I_{24} + I_{23}) \\
&\quad - q_1 q_2 t_3 t_4 (I_1 + I_2) - q_1 t_2 q_3 t_4 (I_1 + I_3) - q_1 t_2 t_3 q_4 (I_1 + I_4) \\
&\quad - t_1 q_2 q_3 t_4 (I_2 + I_3) - t_1 q_2 t_3 q_4 (I_2 + I_4) - t_1 t_2 q_3 q_4 (I_3 + I_4) \\
&\quad - q_1 q_2 q_3 q_4 (I_{123} + I_{134} + I_{124} + I_{234}).
\end{aligned}$$

Call X_2 the similar sum of terms in $a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11}$. We find

$$\begin{aligned}
X_2 &= -q_1 q_2 q_3 q_4 (I_{12} + I_{13} + I_{14} + I_{23} + I_{24} + I_{34}) + q_1 q_2 t_3 q_4 (I_1 + I_2 + I_4) + q_1 q_2 q_3 t_4 (I_1 + I_2 + I_3) \\
&\quad + q_1 t_2 q_3 q_4 (I_1 + I_3 + I_4) + t_1 q_2 q_3 q_4 (I_2 + I_3 + I_4).
\end{aligned}$$

Call finally X_1 the similar sum of terms in $a_2 + a_3 + a_4 + a_5$. We find

$$X_1 = -q_1 q_2 q_3 q_4 (I_1 + I_2 + I_3 + I_4).$$

Summing $X_1 + X_2 - X_3$ we find that all terms with the product $q_1 q_2 q_3 q_4$ cancel term a_1 and the rest terms give in the sum

$$\begin{aligned}
&q_1 q_2 q_3 t_4 (I_3 + I_2 + I_1 + I_{23} + I_{13} + I_{12}) + q_1 q_2 t_3 q_4 (I_1 + I_2 + I_4 + I_{13} + I_{24} + I_{12}) \\
&+ q_1 t_2 q_3 q_4 (I_3 + I_4 + I_1 + I_{14} + I_{34} + I_{13}) + t_1 q_2 q_3 q_4 (I_2 + I_3 + I_4 + I_{23} + I_{24} + I_{34}) \\
&- q_1 q_2 t_3 t_4 (I_1 + I_2) - q_1 t_2 q_3 t_4 (I_1 + I_3) - q_1 t_2 t_3 q_4 (I_1 + I_4) \\
&- t_1 q_2 q_3 t_4 (I_2 + I_3) - t_1 q_2 t_3 q_4 (I_2 + I_4) - t_1 t_2 q_3 q_4 (I_3 + I_4).
\end{aligned}$$

These terms are to be summed with those in a_2 – a_{15} which do not contain D_0 . In the sum we get

$$\begin{aligned}
\Gamma_{RR \rightarrow RRP} &= t_1 t_2 t_3 t_4 D_0 - q_1 q_2 q_3 t_4 I_{123} - q_1 q_2 t_3 q_4 I_{124} - q_1 t_2 q_3 q_4 I_{134} - t_1 q_2 q_3 q_4 I_{234} \\
&\quad + q_1 q_2 t_3 t_4 I_{12} + q_1 t_2 q_3 t_4 I_{13} + q_1 t_2 t_3 q_4 I_{14} + t_1 q_2 q_3 t_4 I_{23} + t_1 q_2 t_3 q_4 I_{24} + t_1 t_2 q_3 q_4 I_{34} \\
&\quad - q_1 t_2 t_3 t_4 I_1 - t_1 q_2 t_3 t_4 I_2 - t_1 t_2 q_3 t_4 I_3 - t_1 t_2 t_3 q_4 I_4. \tag{51}
\end{aligned}$$

As in the previous section, all indefinite expressions like $(tn^-)/Q_-$ at $Q_- = 0$ are to be replaced by $(qn^-)/Q_-$.

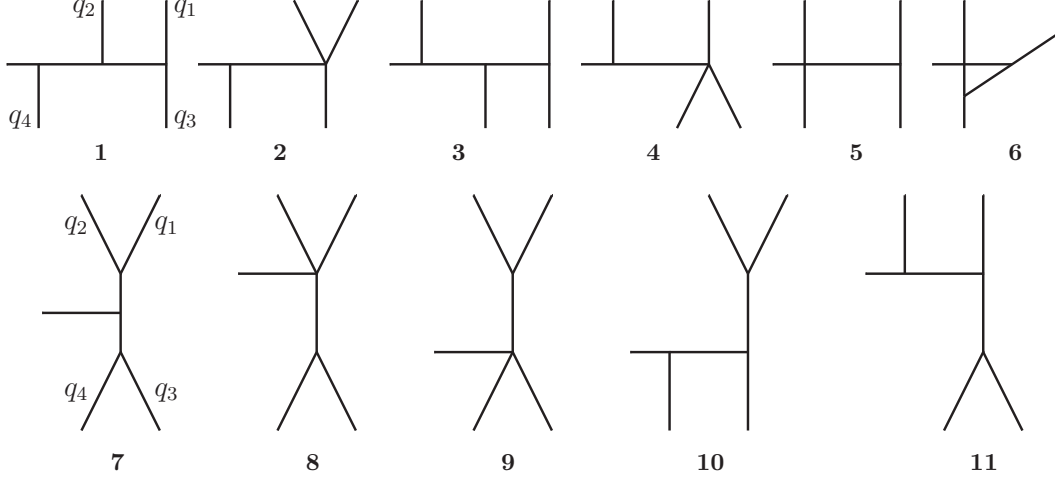


Figure 4: Diagrams for the vertex $RR \rightarrow RRP$ with QCD vertices. Simple external lines directed upwards and downwards actually refer to reggeons.

5.2 Diagrams

All relevant diagrams are shown in Figs. 4-7. The QCD diagrams which correspond to term D_0 are shown in Fig. 4. Diagrams with one induced vertex I_i , $i = 1, 2, 3, 4$ are presented in Fig. 5. Those with two induced vertices I_{ik} , $i < k$ are shown in Fig. 6. Finally, Fig. 7 shows diagrams I_{ikl} , $i < k < l$ with three induced vertices.

Note that from the 11 QCD diagrams shown in Fig. 4 only 6 in the upper part give contribution to the vertex in the original expression (40). Those in the lower line give zero due to the property of the 3-gluon vertex with two of its legs with reggeon momenta. Likewise from the total 102 induced diagrams only 36 take part in the original expression (40). From the diagrams with a single induced vertex they are $I_{i,1}, I_{i,3}, I_{i,4}, I_{i,5}, I_{i,6}$ for $i = 1, 2$ and $I_{i,1}, I_{i,2}, I_{i,3}, I_{i,5}, I_{i,6}$ for $i = 3, 4$ (20 diagrams in all). From the diagrams with two induced vertices they are

$$I_{12,2}, I_{12,5}, I_{23,1}, I_{23,2}, I_{23,4}, I_{23,5}, I_{34,1}, I_{34,5}, \\ I_{14,1}, I_{14,3}, I_{14,4}, I_{14,5}, I_{24,1}, I_{24,3}$$

(14 diagrams in all). And from the diagrams with three induced vertices two diagrams $I_{234,1}$ and $I_{124,2}$. All the rest give zero after multiplication by $n_1 n_2 n_3 n_4$. So the total number of non-zero diagrams in the original expression (40) is 42.

Expression (51) obtained after application of the Ward identities contains all 11 QCD diagrams and additional contributions coming from the induced diagrams. Many of them give zero contributions to (51). Similarly to Fig. 2 they are marked with index (0) in Figs. 5-7. The total number of induced diagrams which give non-zero contributions turns out to be 48. So the use of Ward identities leads to 59 non-zero diagrams, which is 17 diagrams more than the original expression. Still this number can be diminished after the choice of one of the gauges with either $e_+ = 0$ or $e_- = 0$. As in Fig. 2 we mark by indices (\pm) those diagrams in which the leg corresponding to the emitted real gluon carries factor n^\pm . Clearly diagrams marked with (\pm) will vanish in the gauge $e_\pm = 0$. The number of such diagrams with a fixed sort of n is 12. This reduces the number of additional diagrams to 36 and the total number to 47.

We are not going to write out the numerous additional contributions to (51) coming from the induced diagrams here. They are too many and their derivation is straightforward and clear from the similar derivation for the vertex $RR \rightarrow RP$ in the previous section. We only

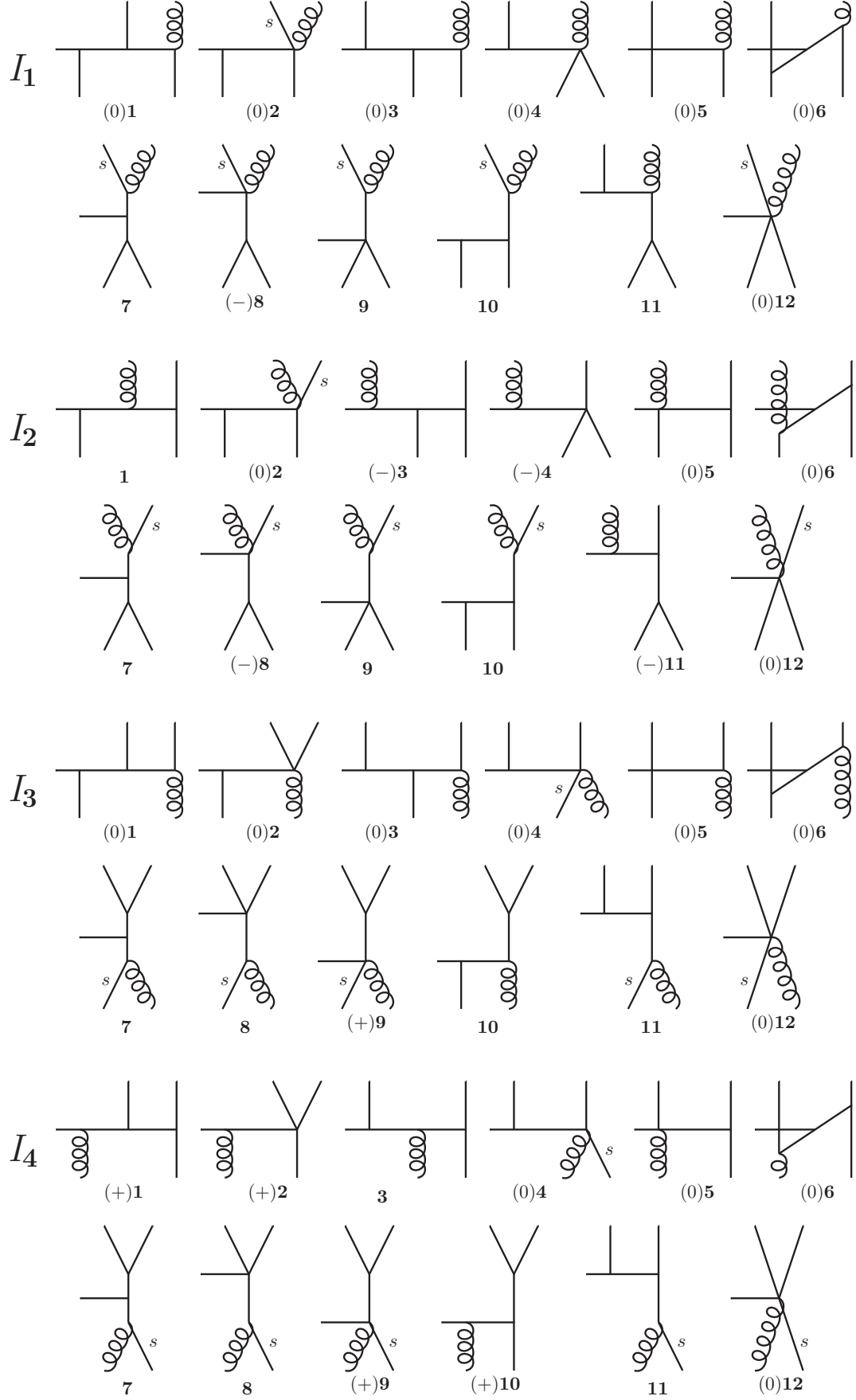


Figure 5: Diagrams for the vertex $RR \rightarrow RRP$ and Ward identities with one induced vertex. Simple external lines directed upwards and downwards actually refer to reggeons.

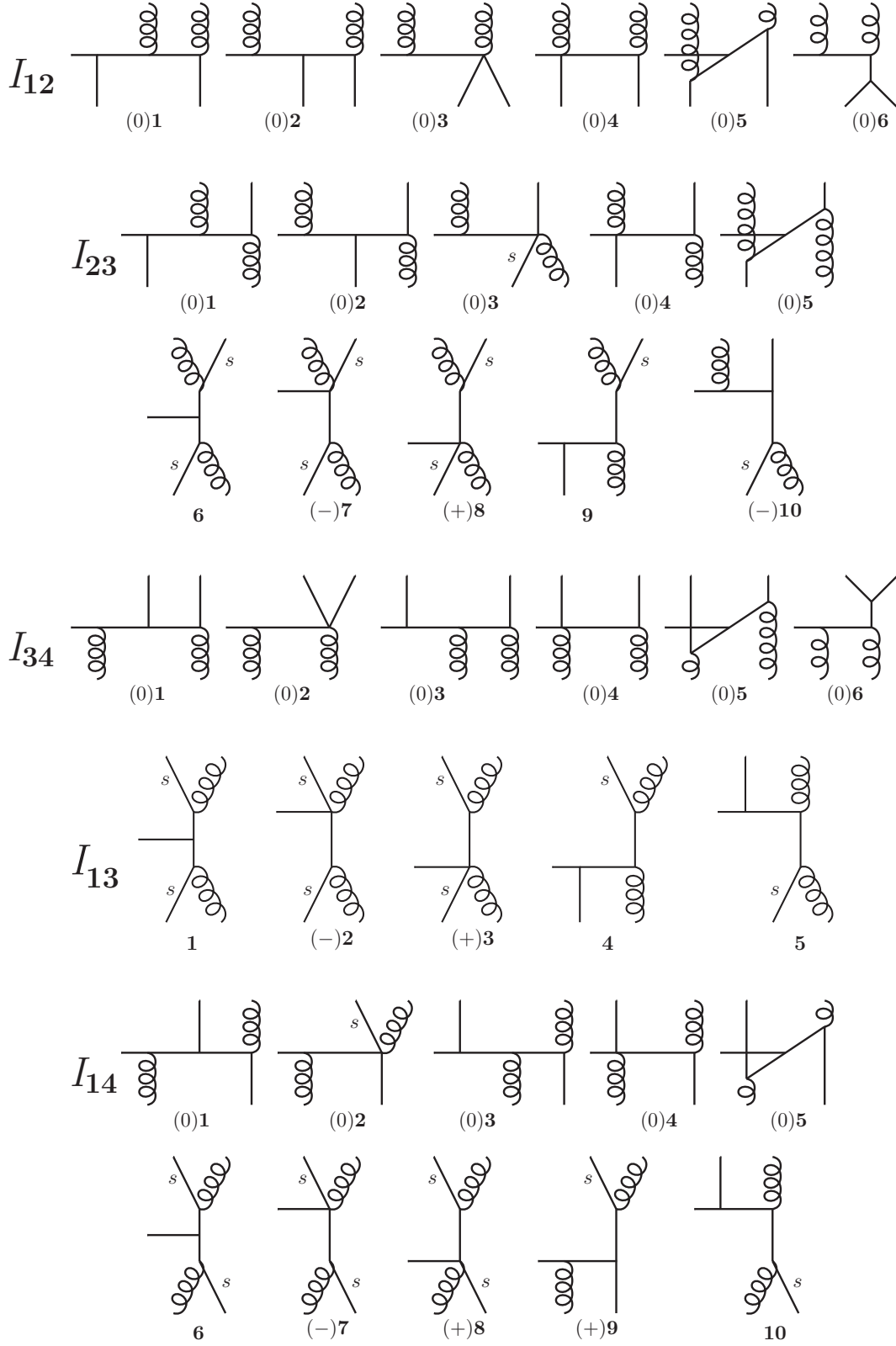


Figure 6: Diagrams for the vertex $RR \rightarrow RRP$ and Ward identities with two induced vertices. Simple external lines directed upwards and downwards actually refer to reggeons.

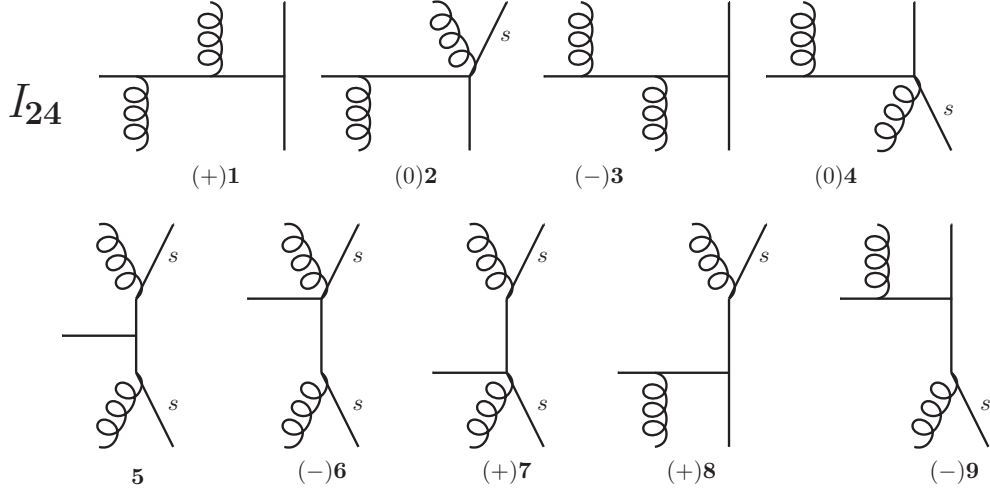


Figure 6: Diagrams for the vertex $RR \rightarrow RRP$ and Ward identities with two induced vertices (continuation).

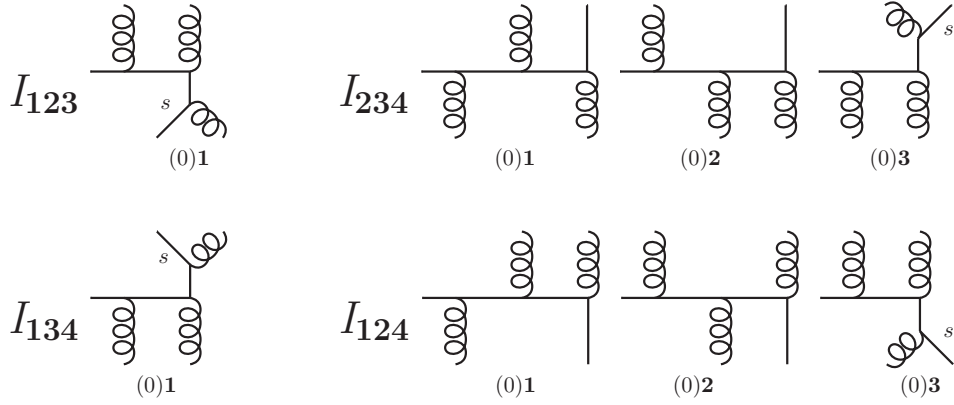


Figure 7: Diagrams for the vertex $RR \rightarrow RRP$ and Ward identities with three induced vertices. Simple external lines directed upwards and downwards actually refer to reggeons.

want to mention, that although the whole set of induced diagrams apart from the vertices V_1, \dots, V_4 , Eq. (10) contains the vertex for transition of a reggeon in 4 gluons, the diagrams with this latter vertex give no contribution to the final expression (51). So we do not need the explicit form of this vertex (which can easily be found using the recurrent relation in [7]).

The obtained contributions have naturally to be symmetrized in ingoing reggeons (with momenta q_1 and q_2) and in outgoing reggeons (q_3 and q_4).

6 Conclusions

Following the suggestion in [9] we studied Ward identities for effective theory of interacting gluons and reggeons in general configurations for vertices $RR \rightarrow RP$ and $RR \rightarrow RRP$. For these Ward identities we have found that though the set of initial reggeon diagrams is indeed to be reduced to exclude transition of a particular reggeon in a single gluon, many more new diagrams are to be included corresponding to the replacement of this reggeon by the gluon. We have found the final expressions which express the vertices via the QCD diagrams convoluted with the transverse momenta plus a certain number of additional diagrams, coming from the induced ones. It turns out that among these additional diagrams there appear singular ones

poorly defined in the Regge kinematics. We formulated the rule to fix their definition.

In the end in the general gauge the final number of diagrams in the expression obtained by means of Ward identities turns out to be larger than in the original expression: 59 against 42 for the vertex $RR \rightarrow RRP$. Use of special gauges allows to reduce the former number to 47: still 5 diagrams more than in the original expression. The number of additional diagrams may be further reduced in special colour configurations, such as the one for the odderon, considered in [10].

Note also that presence of singular diagrams casts doubts on the possibility to get better convergence in the longitudinal momenta applying Ward identities, since in the additional terms which come from them transverse momenta are to be replaced by the total ones.

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